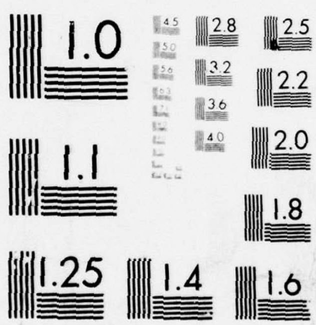
AD-A072 522 HARVARD UNIV CAMBRIDGE MA CENTER ON DECISION AND CON--ETC F/G 12/1
THE GARBLING DECISION MAKER: A MODEL OF BOUNDED RATIONALITY.(U)
MAY 79 N PAPADOPOULOS N00014-77-C-0533

UNCLASSIFIED TR-5 NL

1 OF 1
AD
A072522

END
DATE
FILMED
9-79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

LEVEL II

AD A072522

CENTER ON DECISION AND CONFLICT IN COMPLEX ORGANIZATIONS

VE RI TAS

HARVARD UNIVERSITY

(1)

(6) THE GARBLING DECISION MAKER:

A MODEL OF BOUNDED RATIONALITY

by

(10) Nicholas/Papadopoulos

(11) May 79 (12) 15 p.

(14) TR-5

(9) Technical Report No. 5

Prepared under Contract No. (15) N00014-77-C-0533
Project No. NR 277-240
for the Office of Naval Research

Harvard University
Littauer #309
Cambridge, Massachusetts 02138

May, 1979

The Garbling Decision Maker: A Model of Bounded Rationality

Team theory was developed to model organizations. However, certain problems occur if the theory is applied to the organization designer's problem. It has been demonstrated that increasing information to the agents in a team can only increase the payoff to the team; hence the optimal organization is one where everyone knows everything. This result may be tempered by adding a cost of organization to the team payoff. If the team payoff is augmented by a cost of organization, then the result that the optimal organization is one in which everyone knows everything no longer obtains.

Still, even with this augmented performance index, team theory does not appear to give a realistic model of the organization designer's problem. Team theory traditionally assumes that the decision maker is capable of any manipulation, no matter how complex, on the information that he uses to reach his decision. This ability is not evident in real-life; indeed a very real constraint in the design of organizations is the capability of the agents to process the information given to them. This inability to perfectly process all information received is termed "bounded rationality". The term implies that the decision maker behaves rationally but is only capable of a certain amount of manipulation.

In this paper, a general model is presented for a team with boundedly rational decision makers. The decision maker is modelled as one that makes the correct decision, but garbles that decision at a rate dependent on the amount of information he processes.

A Linear-Quadratic-Gaussian model is presented based on this general model. A result is demonstrated which shows that no descriptive gain is obtained by using this model of the decision maker, when a quadratic payoff is used. Finally, the model of the decision maker is applied to a problem in returns to scale under uncertainty. The usual model of informational cost yields increasing returns to scale when information is used; the model with the garbling decision maker yields constant returns to scale.

1. General Model

The general model of the problem of finding the optimal organization can be formulated as finding the highest mean reward over possible organizations and decision rules:

$$\max_{\substack{\text{organizations} \\ \text{decision rules}}} E\, R(\underline{x},\underline{y}) - \text{cost of organization}$$

$\underline{x}$ are input symbols available to the organization

$\underline{y}$ are output symbols (controls) produced by the organization

R(,) is the reward function -- the reward given to organization when it yields the second argument when the first argument is available

the expectation is taken over the input symbols whose probability distribution is assumed.

The organization produces output symbols from input symbols by employing agents who read input or intermediate signals and produce intermediate or output signals. The decision rules are simply the transformation rules employed by the agents. The organization is specified by which signals are read and written by which agents. This model of the organization is essentially the team theoretic formulation of the decision problem. The main difference in this model is the model of the boundedly rational decison maker.

The decision maker may be told that the optimal rule is to map x_j into y_k, but he will tend to make errors and the transformation implemented will actually map x_j into y_k with a certain probability $p(y_k/x_j)$ and map x_j into other symbols with probability $1 - p(y_k/x_j)$. The probability that the correct transformation is made- $p(y_k/x_j)$-- is denoted <u>accuracy</u> and is inversely related to the <u>complexity</u> of the input symbols used to make the decision. Complexity is a purposefully vague term which is supposed to measure the informational load placed on one agent by the input signal read by that agent. The complexity function is sometimes denoted C() and may be taken to be the Shannon informational entropy of the input signal.

The cost of the organization noted above may be taken to be the costs of the agents (salaries &c.) and the costs of the communications involved.

It is assumed that the input symbols and outputs symbols are multidimensional so that agents may read or write only parts of the symbols. Thus a component of the x given to the organization may correspond to a specific output--say a marketing decision on a certain product.

Remarks on the model

The model of the organization is essentially that of R. R. Drenick (1976). The model presented above differs from Drenick's in the following ways:

1) The problem of satisfying a constraint on the mean time to complete tasks is ignored;

2) The dependence on psychological factors of agent performance is ignored;

3) The nature of the input/output symbols have explicitly been made multidimensional so that the study of coordination versus informational overload can be more easily accomplished.

The specification of the input alphabets, output alphabets, the reward structure, the sources and destinations of the symbols (signals) and the rules of transformation comprise a complete description of the organization. In the language of nonclassical control, input symbols correspond to information, intermediate signals to communication among decision makers, output symbols to controls, reward function to payoff function, sources and destinations of symbols to information and communication patterns, and the rules of transformation correspond to decision rules.

The key feature of this model is the complexity-accuracy relation. This relation is intended to capture the concept of rationality by making the decision maker less effective when he is overloaded with information. Presumably, when the decision maker is completely overloaded, he will act as a random source, independent of the input given him, making him useless to the organization. Another viewpoint, and one which resolves a bit of ambiguity, is the following decomposition of the boundedly rational decision maker into two parts. The first part of the decision maker makes the team optimal decision. The second part of the decision maker is a noisy communication channel that takes the output of the first part and garbles it to produce the actual output.

The problem, as formulated above, is, given the input and output alphabets, the reward function and the characteristics of the decision makers, design the organization so as to maximize the mean reward. "Design the organization" means to specify the communication links--information sets--and the transformation rules--strategies. One may also ask for the appropriate design of the input and output alphabets; this question is equivalent to designing the basic information sets and the coordinates of the controls. By asking this latter question, one may gain insight into the problem where fixed input and output alphabets are given.

2. LQG Static Team Formulation

In this section the Linear-Quadratic-Gaussian Static Team formulation is explored. The assumptions that the information is a <u>linear</u> function of the underlying random state that has a <u>gaussian</u> distribution and that the payoff function is <u>quadratic</u> characgerize this formulation. Also, the <u>static</u> nature of the information sets implies that there is no communication or signalling between agents; the team assumption means that all agents have the same objective function. This formulation is traditionally used in non-classical control theory as a first step to understanding a particular problem; this is the use that the formulation is put to here.

Formulation

The objective function is

$$E\ \underline{u}'Q\underline{u} + 2\ \underline{u}'S\underline{x} \qquad (2.1)$$

$\underline{u}$ is the control vector.

Player i controls u_i.

Q is taken to be negative definite.

The random variables in the problem are (2.2)

$\underline{x}$ k-dimensional underlying state $\sim N(0,I_k)$

$\underline{v}$ m-dimensional measurement noise, independent of x $\sim N(0,I_m)$

$\underline{w}$ ℓ-dimensional inaccuracy noise, independent of x & v $\sim N\{0,\mathrm{Diag}\{f_1(C(\theta_1)),\ldots,f_\ell(C(\theta_\ell))\}\}$

There are m pieces of information available:

$$z^j = h^j x + v_j \qquad (2.3)$$

The control is given as a function of the decision maker's information plus additive white noise corresponding to the inaccuracy induced by the informational loading.

$$u_i = \gamma_i(\theta_i) + w_i \qquad (2.4)$$

The notation for the individual information sets is designed to allow completely arbitrary information structures. n_i denotes the number of informations read by the i^{th} decision maker. So the information set for the i^{th} decision maker is

$$\theta_i = (z^{s(i,1)},\ldots,z^{s(i,n_i)})' = H_i x + E^i \underline{v} \qquad (2.5)$$

$$H_i = (h^{s(i,1)'},\ldots,h^{s(i,n_i)'})' \qquad (2.6a)$$

$$E^{i}_{jk} = \{1 \text{ if } k = s(i,j);\ 0 \text{ otherwise}\} \qquad (2.6b)$$

The definitions of H_i and E^i are mainly for calculational convenience and serve to emphasize the linearity of the resulting information structure.

A few functions have been used in the above definitions and are defined as follows:

$\gamma_i(\ )$ the optimal strategy for the i^{th} decision maker as a function of his information set.

$f_i(\ )$ the accuracy-complexity relation----- the variance of the noise added to the optimal team decision.

$C(\ )$ the complexity of the information set--it is independent of the realization of the information set. If Shannon informational entropy is used then $C(\theta_i) = 1/2\ n_i \ln 2\pi\varepsilon \quad + 1/2 \ln |H_i H_i' + I|$

Separation Result

A separation result is shown which permits easy calculation of the optimal strategies and costs for a given information structure.

Proposition. The optimal strategies γ_i for the team problem as stated above can be computed as if $\underline{w} \equiv 0$. Furthermore, the optimal payoff is the sum of the usual team payoff ($\underline{w} \equiv 0$) and the weighted sum of the $f_i\{C(\theta_i)\}$

Proof. Let $\Gamma = (\gamma_1(\theta_1),\ldots,\gamma_\ell(\theta_\ell))'$. Then substituting the controller definition (3.4) into the payoff (3.1) yields:

$$\max E\{(\Gamma + \underline{w})'Q(\Gamma + \underline{w}) + 2(\Gamma + \underline{w})'Sx)$$

Rewriting this payoff using the zero mean and independence properties of w yields

$$\max_{\Gamma} E\ \{\Gamma'Q\Gamma + 2\Gamma'S\underline{x}\} + E\underline{w}'Q\underline{w}\}$$

But the expectation involving the term in braces is simply the usual team payoff ($\underline{w}\equiv 0$), $E\underline{w}'Q\underline{w}$ is independent of Γ and is equal to $\Sigma_i\{q_{ii}f_i\ C(\theta_i)\}$ qed

Remarks.

First note that the properties of linear information and gaussian distribution were not used in the proof. Only the properties that the inaccuracy noise was independent of the control (state variables) and additive, and that the payoff was quadratic, were used.

Secondly note that this result allows easy calculation of the optimal decisions. One may use the usual team decision rules that have been well developed.

Thirdly note that this result implies that no descriptive gain has been made by considering the use of a garbling decision maker in the context of the quadratic payoff. Since the f_i and C( ) functions are arbitrary, the total team payoff may be written as

$$E(\underline{u}'Q\underline{u} + 2\ \underline{u}'S\underline{x}) + \text{cost of organization.}$$

This payoff is the usual payoff given in team theory for the design of organizations. This formulation does have the advantage of giving another interpretation to the additive cost of organization in the quadratic payoff context: cost of informational loading. In this context another interpretation can be given to the organization designer's problem. When he considers increasing information to agents in the organization, he must balance the effect of increasing the team payoff by adding information versus the effect of decreasing the team payoff by adding to the costs of inforamtional loading. The fact that the increased cost due to informational loading reduces to the usual additive cost is disappointing because it does not change the mathematical form of the designer's problem. This reduction to the usual form may be viewed as a degeneracy and may be taken as another argument against the use of quadratic payoff functions. This reduction does not appear in all cases; a useful example is given in the next section.

3. An Application to Informational Economies of Scale

Wilson (1975) has considered the problem of informational economies of scale. He has shown that certain constant returns to scale uncertain production functions exhibit increasing returns to scale in information. He has shown that a firm operating with one of these technologies will desire to produce at infinite scale even if the firm is risk averse. By applying the model of the garbling decision maker and the separation result to one of Wilson's models, it can be demonstrated that constant returns to scale in information is obtained. As a direct consequence, the firm experiences declining utility in scale if it is risk averse.

Wilson considers the stochastic technology

$$y = ER(1 - (a - \theta)^2) \tag{3.1}$$

where R is the scale
a is the firms decision for process parameter
θ is the state of nature.

Wilson then argues that the organization designer's problem may be written as the cost of organization plus the usual payoff:

$$\max_z ER(1 - (a(z) - \theta)^2) - c(z)) \tag{3.2}$$

where c(z) is the cost of information structure
z is the information structure.

Wilson then argues that there are increasing returns to scale in information. The argument is essentially that the benefits of information are multiplied by the scale R, but that the cost of information is the same regardless of scale. Hence, the firm will wish to produce at infinite scale. He shows that the same result applies if the firm is risk averse.

If instead of this cost of information model, one uses a garbling decision maker, the following results obtain. The organization designer's problem is

$$\max_z ER(1 - (a(z) + w(C(z)) - \theta)^2) \tag{3.3}$$

where w is the garbling noise and has
variance f(C(z)), where C(z) measures
the complexity of the information structure.

Note that by applying the separation result, or by straightforward manipulation, the designer's problem becomes

$$\max_z ER[1 - (a(z) - \theta)^2 - f(C(z))]. \tag{3.4}$$

Now informational economies of scale are constant since R multiplies both the cost and benefit of information. Of course, if the firm is risk averse, there will eventually be decreasing utility to scale and there may exist an optimal scale.

Note again that these results apply quite generally; recall that the proof of the separation theorem, and hence the proof of constant returns to scale in information, depend only on the payoff structure and the independence of the garbling noise with respect to other random variables in the problem.

4. Conclusion

A reasonable and plausible model for the boundedly rational decison maker has been presented. The decision maker reacts to increasing informational load by garbling his optimal decision. In the quadratic case, this garbling appears as an additive organizational cost. The implication of this cost is two-fold. First, another interpretation can be given to the additive organizational cost: informational loading on the decision makers. Second, the mathematical form of the organization designer's problem remains the same; there are no new conclusions that this formulation yields over the usual formulation for the quadratic payoff. A problem has been presented where the form is changed and a new conclusion obtains. Where a traditional formulation leads to increasing informational economies of scale; this formulation leads to constant economies of scale, a much more plausible outcome.

References

Drenick, R.F. Organization and Control in Ho & Mitter Directions in Large Scale Systems, (Plenum 1976), pages 279-302.

Wilson, R. "Informational Economics of Scale", Bell Journal of Economics, Spring 1975, pages. 184-195.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Center on Decision and Conflict in Complex Organizations	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

THE GARBLING DECISION MAKER: A MODEL OF BOUNDED RATIONALITY

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report No. 5

5. AUTHOR(S) (First name, middle initial, last name)

Nicholas Papadopoulos

6. REPORT DATE May, 1979	7a. TOTAL NO. OF PAGES 12	7b. NO. OF REFS 2
8a. CONTRACT OR GRANT NO. N00014-77-C-0533 b. PROJECT NO. NR-277-240 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report #5 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its publication is unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Logistics and Mathematics Statistics Branch, Department of the Navy, Office of Naval Research, Wash.D.C.

13. ABSTRACT

A model is constructed of a garbling decision maker -- one who garbles his decision in response to the quantity of information he uses to reach his decision. This model is supposed to capture some of the effects of bounded rationality on decision making. This model of the decision maker is imbedded in a team-model of an organization to model the organization designer's problem. A Linear-Quadratic-Gaussian static team formulation is given and a separation result is proven that shows that the form of the problem reduced to the usual formulation of adding the cost of organization to the team objective, giving a new interpretation to the traditional formulation. While this result is uninteresting generally and is an artifact of the quadratic payoff, it may be used to show constant informational returns to scale in a case where a more usual cost of information yields increasing informational returns to scale.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Unclassified

Security Classification

3ND PPSO 13152